# Stereo Matching Using Belief Propagation

*ISL Lab Seminar*

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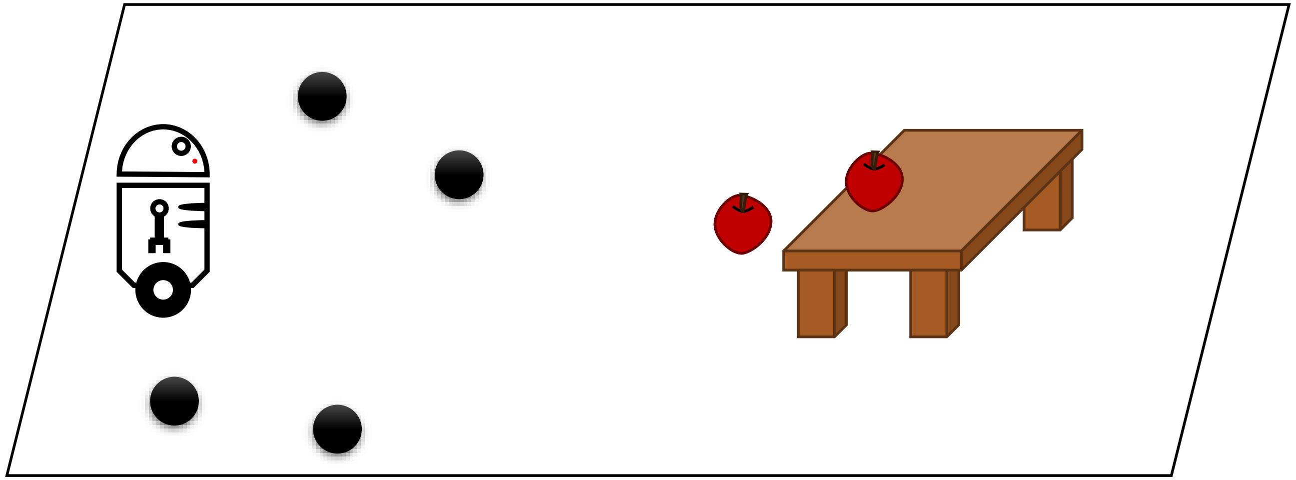
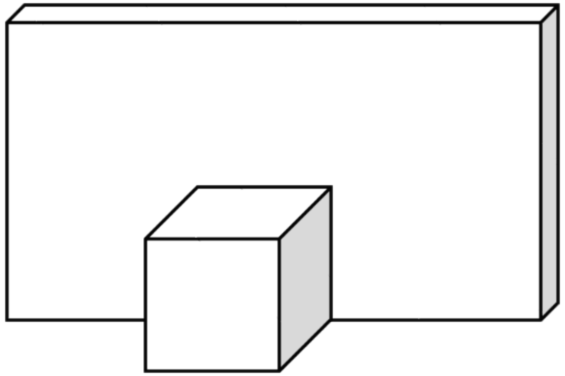
# Contents

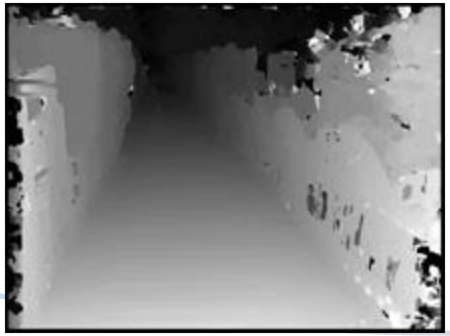
# Introduction

## ❖ Stereo Vision

Robot navigation, 3D reconstruction…



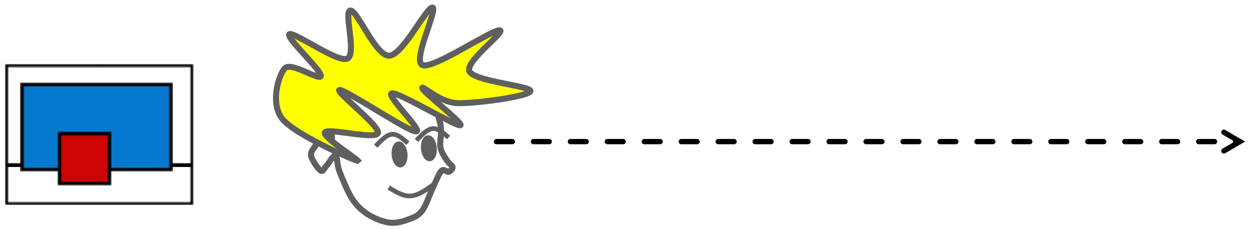


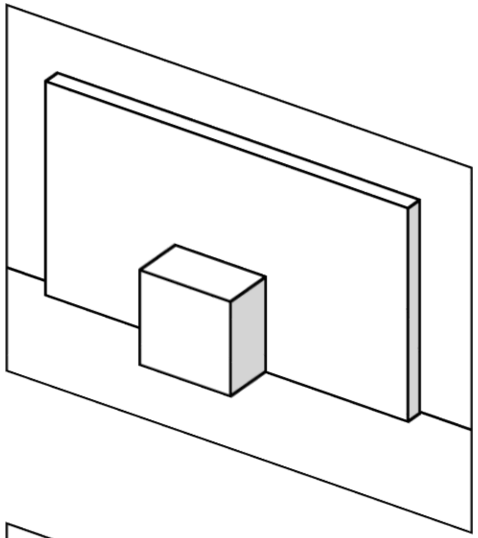


# Introduction

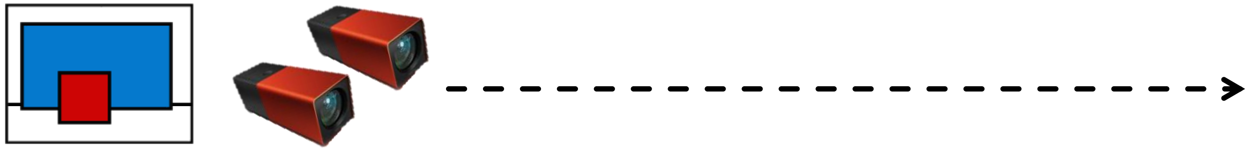
## ❖ Stereo Vision

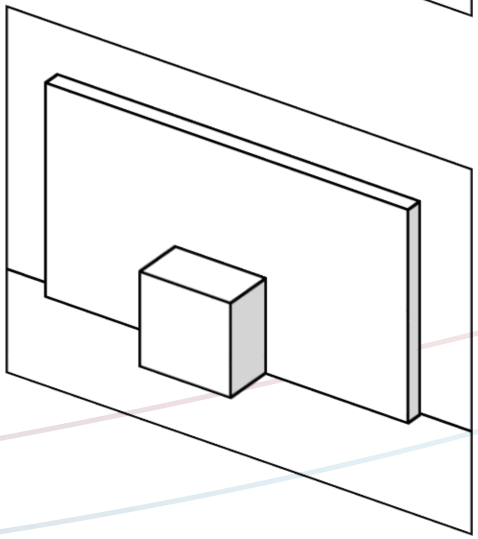
**Human's eye**





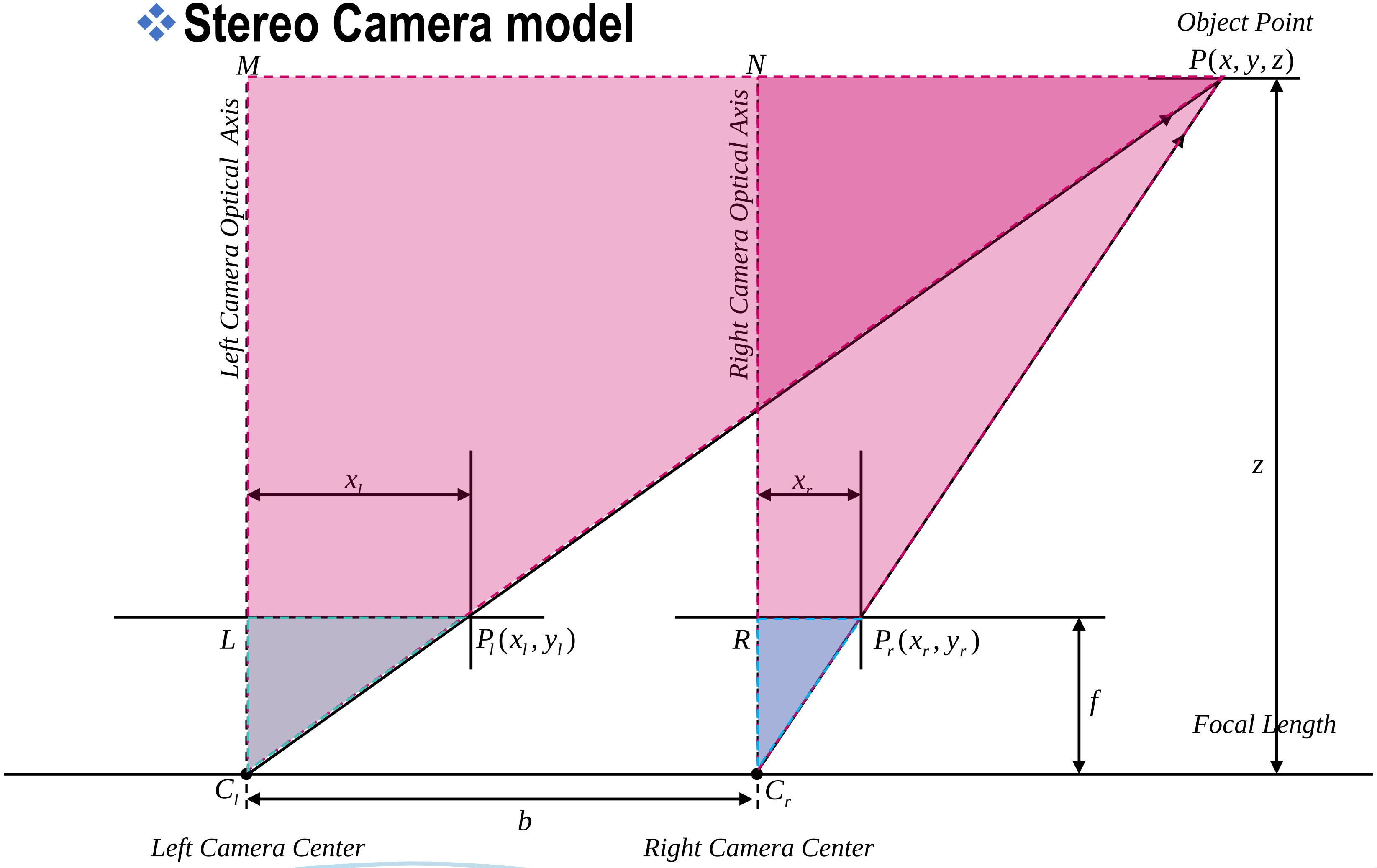
**Stereo Camera**





# Stereo Vision

## ❖ Stereo Camera model




$$\Delta PMC_l \overset{similar}{\longleftrightarrow} \Delta P_l LC_l$$

$$\frac{x}{z} = \frac{x_l}{f} \qquad \text{——— a}$$

$$\Delta PNC_r \overset{similar}{\longleftrightarrow} \Delta P_r RC_r$$

$$\frac{x-b}{z} = \frac{x_r}{f} \qquad \text{——— b}$$

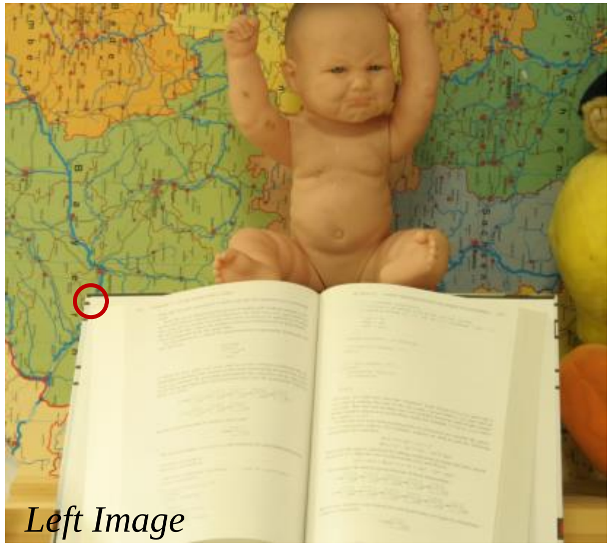
from a $x = \frac{x_l}{f} z$ from b $x = \frac{x_r}{f} z + b$

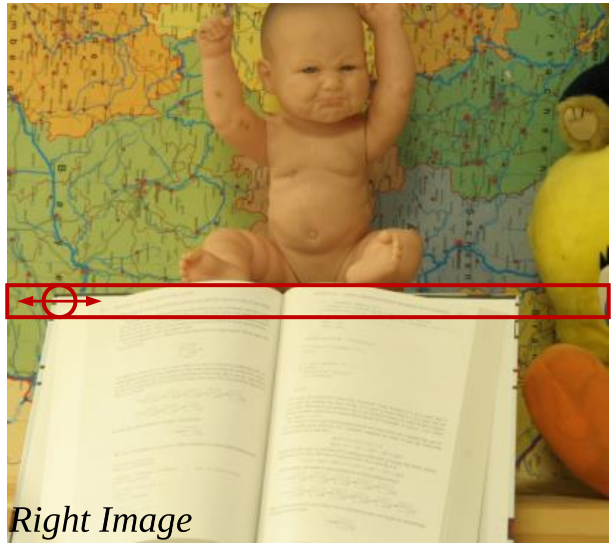
$$\frac{x_l}{f} z = \frac{x_r}{f} z + b, \quad \frac{x_l - x_r}{f} z = b$$

$$\therefore z = \frac{bf}{x_l - x_r}$$

# Stereo Vision

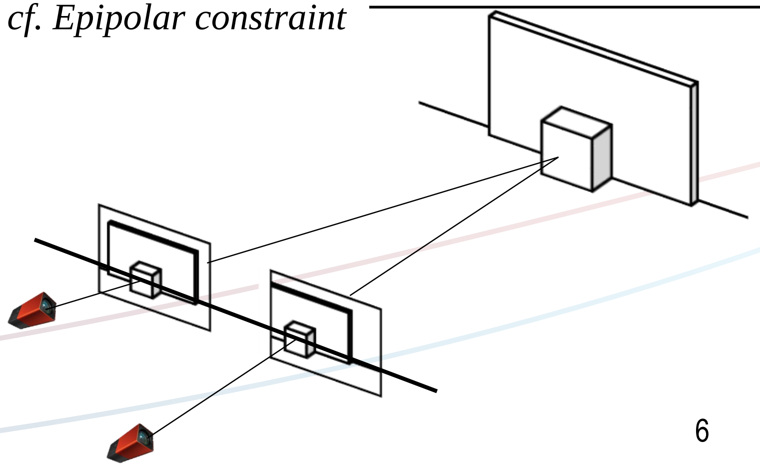
## ❖ Correspondence problem




- Noise
- Low-texture region
- Depth-discontinuity
- Occlusion

*cf. Epipolar constraint*



# Stereo Vision

## ❖ Stereo matching method – Local method

SSD (Sum of Squared Difference)

$$SSD_{MN}(x, y, d) = \sum_{y=1}^{M} \sum_{x=1}^{N} [I_l(x, y,) - I_r(x-d, y)]^2$$

SAD (Sum of Absolute Difference)

$$SAD_{MN}(x, y, d) = \sum_{y=1}^{M} \sum_{x=1}^{N} | I_l(x, y,) - I_r(x-d, y) |$$

➡ **Local minima problem**

MAE(Mean Absolute Error)

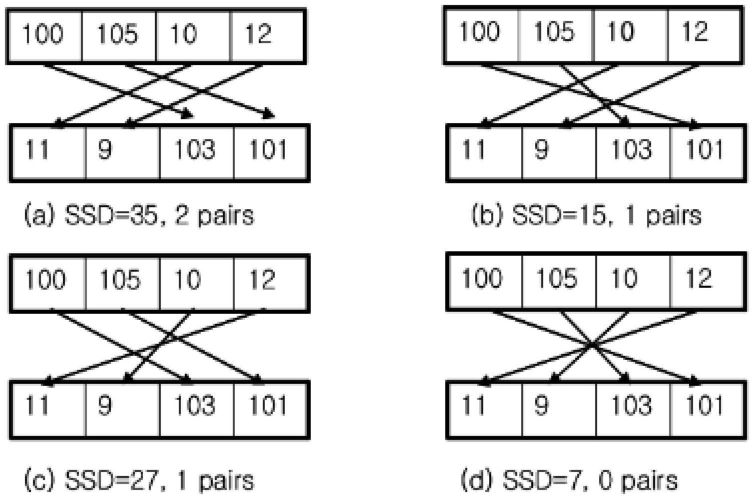
$$MAE_{MN}(x, y, d) = \frac{1}{M \times N} \sum_{y=1}^{M} \sum_{x=1}^{N} | I_l(x, y,) - I_r(x-d, y) |$$

# Stereo Vision

## ❖ Stereo matching method – Global method

Use the energy function

$$E(d) = E_{data}(d) + \lambda E_{smoothness}(d)$$



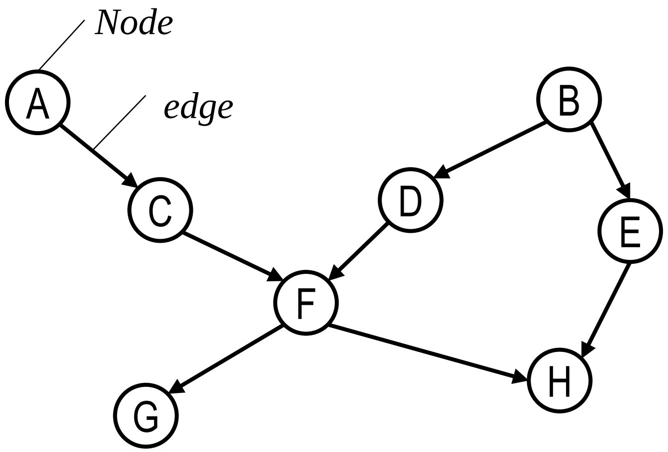

Implementation :

**Belief Propagation(BP)**, Graph Cut(GC), Dynamic Programing(DP)…

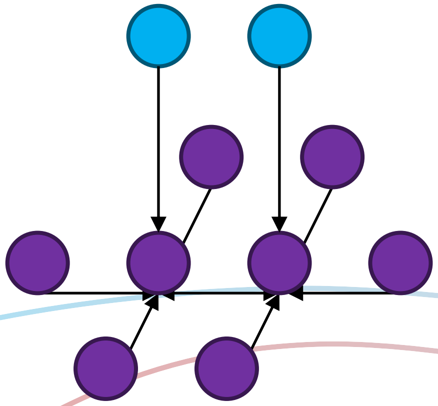
# Belief Propagation

## Bayesian network

: Directed and acyclic




## HMM (Hidden Markov Model)



Observable node(Y) : Image

Hidden node(X) : Disparity

## MRF (Markov Random Field)

: Undirected and cyclic

1) Positivity

$$P(f) > 0, \forall f$$

2) Markovianity

$$P(f_p \mid f_{P-\{p\}}) = P(f_p \mid f_{N_p})$$

# Belief Propagation

## ❖ Goal

: Computes **marginal probability** of hidden nodes

## ❖ Attributes

: **Iterative** algorithm

: **Message passing** between neighboring hidden nodes

## ❖ Procedure

1) Select random neighboring hidden nodes $x_i, x_j$

2) Send message $m_{ij}$ from $x_i$ to $x_j$

3) Update belief about marginal probability at node $x_j$

# Belief Propagation

- Probabilistic Stereo Model


likelihood

posterior

prior (Hypothesis)

$$P(D\,|\,I) = \frac{P(I\,|\,D)P(D)}{P(I)}$$

evidence (Data)

# Belief Propagation

- **Likelihood**

$$P(D|I)=\frac{P(I|D)P(D)}{P(I)}$$

Matching cost function

$$P(I|D)\propto\prod_i \exp(-F(i,d_i))$$

$d_i$ : disparity candidate at pixel i

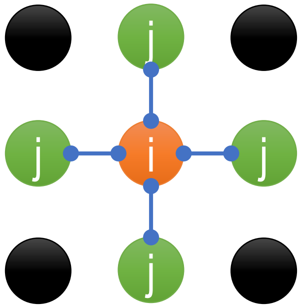
- **Prior**

$$P(D|I)=\frac{P(I|D)P(D)}{P(I)}$$

Constraint function

$$P(D)\propto\prod_i\prod_{j\in N(i)}\exp(-V(d_i,d_j))$$

$$V(d_i,d_j)=|d_i-d_j|$$



# Belief Propagation

## ❖ Probabilistic Stereo Model

$$P(D \mid I) = \frac{P(I \mid D)P(D)}{P(I)}$$

$$\propto P(I \mid D)P(D)$$

$$\propto \prod_i \exp(-F(i, d_i, I)) \prod_i \prod_{j \in N(i)} \exp(-V(d_i, d_j))$$

$$= \prod_i \psi_i(x_i, y_i) \prod_i \prod_{j \in N(i)} \psi_{ij}(x_i, x_j)$$

$\psi_i$ : local evidence for node $x_i$

$\psi_{ij}$ : compatibility matrix between nodes $x_i$ and $y_i$

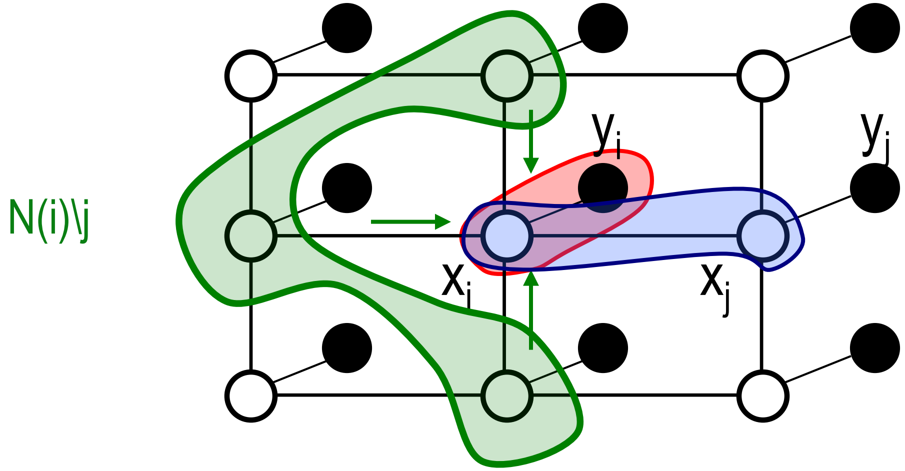
**MAP** ➡ maximize marginal probability (maximize belief)

$$P(x_i) = \sum_{x_1, x_2, \ldots, x_{i-1}} \sum_{x_{i+1}, \ldots, x_N} P(x_1, x_2, \ldots, x_N)$$

# Belief Propagation

## ❖ Message Passing

: Message $m_{ij}$ from $x_i$ to $x_j$



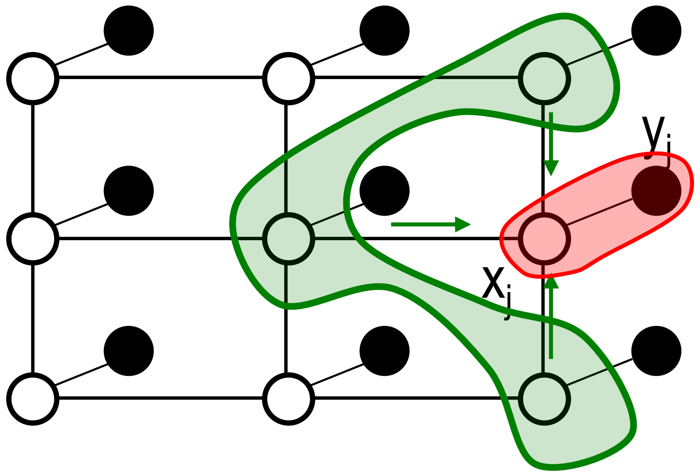

$$m_{ij}(x_j) = \kappa \max_{(xi)} [\psi_i(x_i, y_i)\ \psi_{ij}(x_i, x_j) \prod_{xk \in N(i) \setminus j} m_{ki}(x_i)]$$

$$m_i(x_i) = \psi_i(x_i, y_i)$$ : local evidence

# Belief Propagation

## ❖ Belief Update

: Belief $b(x_j)$




$$b_j(x_j) = \kappa\, \psi_j(x_j, y_j) \prod_{xk \in N(j)} m_{ij}(x_j)$$

# Belief Propagation

## ❖ Implementation of message, belief and disparity

$$m_{ij}^{t+1}(x_j) = \kappa \max_{x_i} \left[ \psi_i(x_i, y_i)\psi_{ij}(x_i, x_j) \prod_{x_k \in N(x_i)\backslash x_j} m_{ki}^t(x_i) \right]$$

$$b_i(x_i) = \kappa \psi_i(x_i, y_i) \prod_{x_k \in N(x_i)\backslash x_j} m_{ki}(x_i)$$

$$d_i^{MAP} = \arg\max_{x_k} b_i(x_k)$$

take the negative logarithm of each equation

$$M_{ij}^{t+1}(x_j) = c \min_{x_i} \left[ M_i(x_i) + \phi_c(x_i, x_j) + \sum_{x_k \in N(x_i)\backslash x_j} M_{ki}^t(x_i) \right]$$

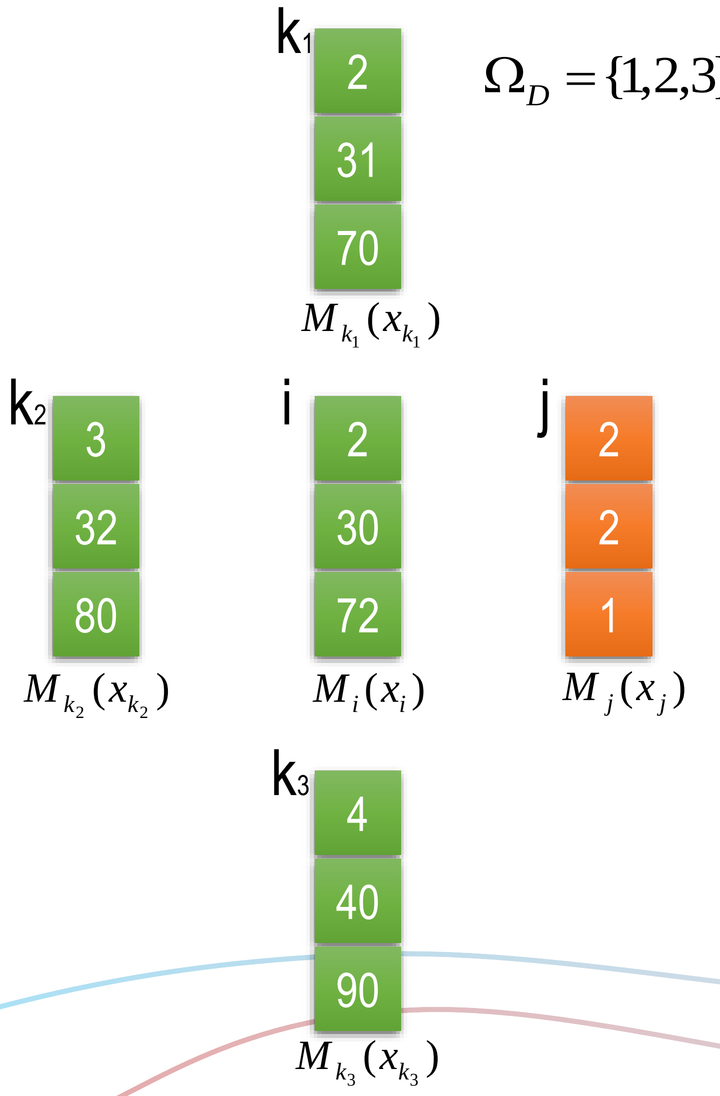
$$B_i(x_i) = c \left[ M_i(x_i) + \sum_{x_k \in N(x_i)\backslash x_j} M_{ki}(x_i) \right]$$

$$D_i^{MAP} = \arg\min_{x_k} b_i(x_k)$$

# Belief Propagation

$$M_{ij}^{t+1}(x_j) = c \min_{x_i}\left[ M_i(x_i) + \phi_c(x_i, x_j) + \sum_{x_k \in N(x_i)\backslash x_j} M_{ki}^t(x_i) \right]$$

## ❖ Example




$$\Omega_D = \{1,2,3\}$$

1) Initialize

$$M_{ij}^0(x_j) = [0 \quad 0 \quad 0]^T$$

$$M_{k_1 i}^0(x_i) = M_{k_2 i}^0(x_i) = M_{k_3 i}^0(x_i) = [0 \quad 0 \quad 0]^T$$

2) Update

$$M_{ij}^1(x_j = 1)$$

$$= \min\begin{pmatrix} \left(0 + 2 + M_{k_1 i}^0(x_i = 1) + M_{k_2 i}^0(x_i = 1) + M_{k_3 i}^0(x_i = 1)\right), \\ \left(1 + 30 + M_{k_1 i}^0(x_i = 2) + M_{k_2 i}^0(x_i = 2) + M_{k_3 i}^0(x_i = 2)\right), \\ \left(2 + 72 + M_{k_1 i}^0(x_i = 3) + M_{k_2 i}^0(x_i = 3) + M_{k_3 i}^0(x_i = 3)\right) \end{pmatrix} = 2$$

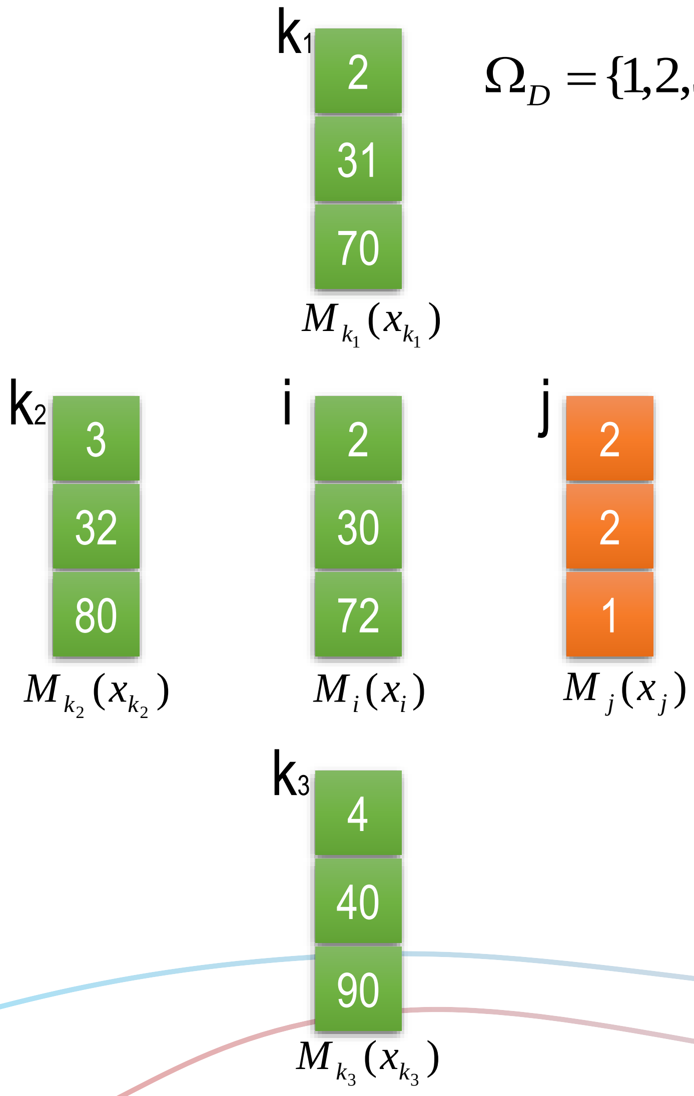
$$M_{ij}^1(x_j = 2)$$

$$= \min\begin{pmatrix} \left(1 + 2 + M_{k_1 i}^0(x_i = 1) + M_{k_2 i}^0(x_i = 1) + M_{k_3 i}^0(x_i = 1)\right), \\ \left(0 + 30 + M_{k_1 i}^0(x_i = 2) + M_{k_2 i}^0(x_i = 2) + M_{k_3 i}^0(x_i = 2)\right), \\ \left(1 + 72 + M_{k_1 i}^0(x_i = 3) + M_{k_2 i}^0(x_i = 3) + M_{k_3 i}^0(x_i = 3)\right) \end{pmatrix} = 3$$

# Belief Propagation

- **Example**

$$M_{ij}^{t+1}(x_j) = c \min_{x_i} \left[ M_i(x_i) + \phi_c(x_i, x_j) + \sum_{x_k \in N(x_i) \backslash x_j} M_{ki}^{t}(x_i) \right]$$




$$\Omega_D = \{1,2,3\}$$

$$M_{ij}^{1}(x_j = 3)$$

$$= \min \begin{pmatrix} \left(2 + 2 + M_{k_1 i}^{0}(x_i = 1) + M_{k_2 i}^{0}(x_i = 1) + M_{k_3 i}^{0}(x_i = 1)\right), \\ \left(1 + 30 + M_{k_1 i}^{0}(x_i = 2) + M_{k_2 i}^{0}(x_i = 2) + M_{k_3 i}^{0}(x_i = 2)\right), \\ \left(0 + 72 + M_{k_1 i}^{0}(x_i = 3) + M_{k_2 i}^{0}(x_i = 3) + M_{k_3 i}^{0}(x_i = 3)\right) \end{pmatrix} = 4$$

$$M_{ij}^{1} = [2 \quad 3 \quad 4]^T$$

$$M_{k_1 i}^{1}(x_i) = [2 \quad 3 \quad 4]^T$$

$$M_{k_2 i}^{1}(x_i) = [2 \quad 3 \quad 4]^T$$

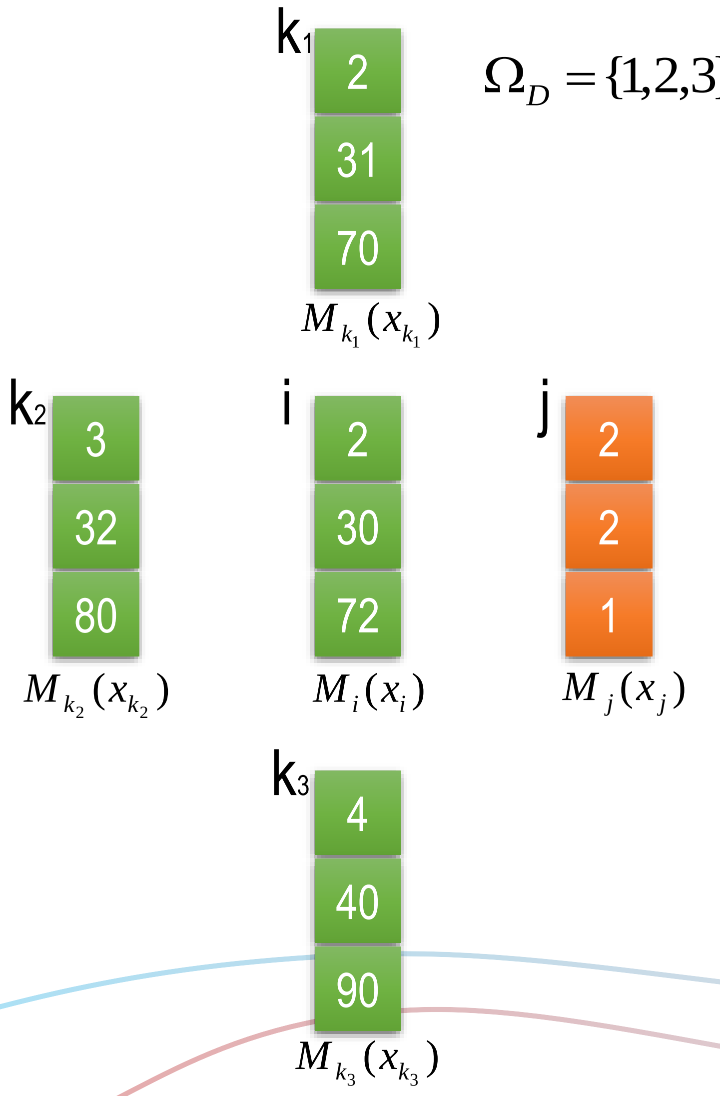
$$M_{k_3 i}^{1}(x_i) = [3 \quad 4 \quad 5]^T$$

$$M_{k_4 i}^{1}(x_i) = [4 \quad 5 \quad 6]^T$$

$$\Rightarrow \quad M_{ij}^{2}(x_j) = [11 \quad 12 \quad 13]^T$$

# Belief Propagation

## ❖Example




$$\Omega_D = \{1,2,3\}$$

$$B_i(x_i) = c\left[ M_i(x_i) + \sum_{x_k \in N(x_i) \setminus x_j} M_{ki}(x_i) \right]$$

$$D_i^{MAP} = \arg\min_{x_k} b_i(x_k)$$

3) Calculate Belief

$$B_j(x_j = 1) = 2 + 11 = 13$$

$$B_j(x_j = 2) = 2 + 12 = 14$$

$$B_j(x_j = 3) = 1 + 13 = 14$$

$$B_j(x_j) = [13 \quad 14 \quad 14]^T,$$

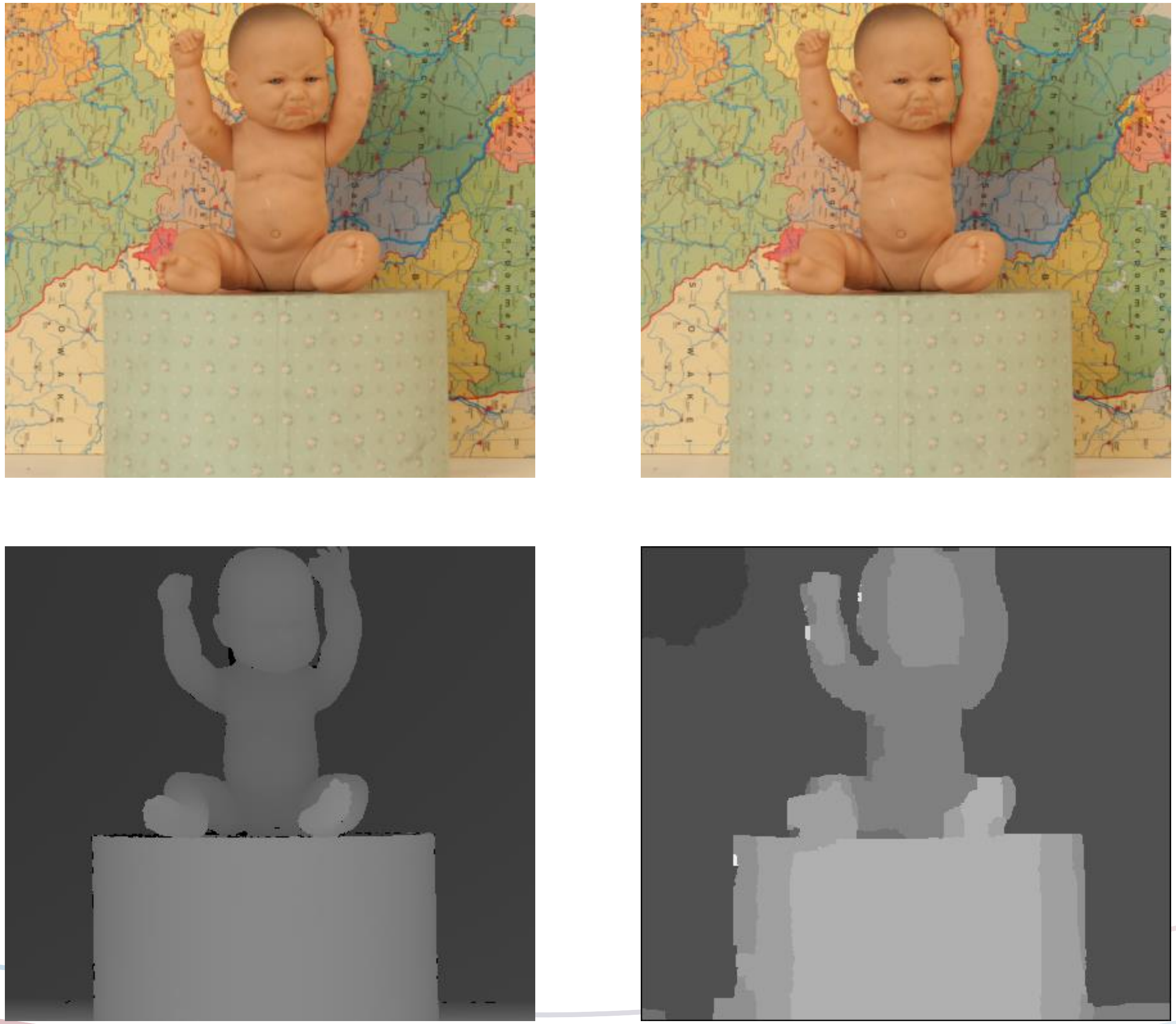
$$x_j^{MAP} = \arg\min_{x_k} B_j(x_k) = 1$$

# Belief Propagation

- Example

# Results

- baby1*

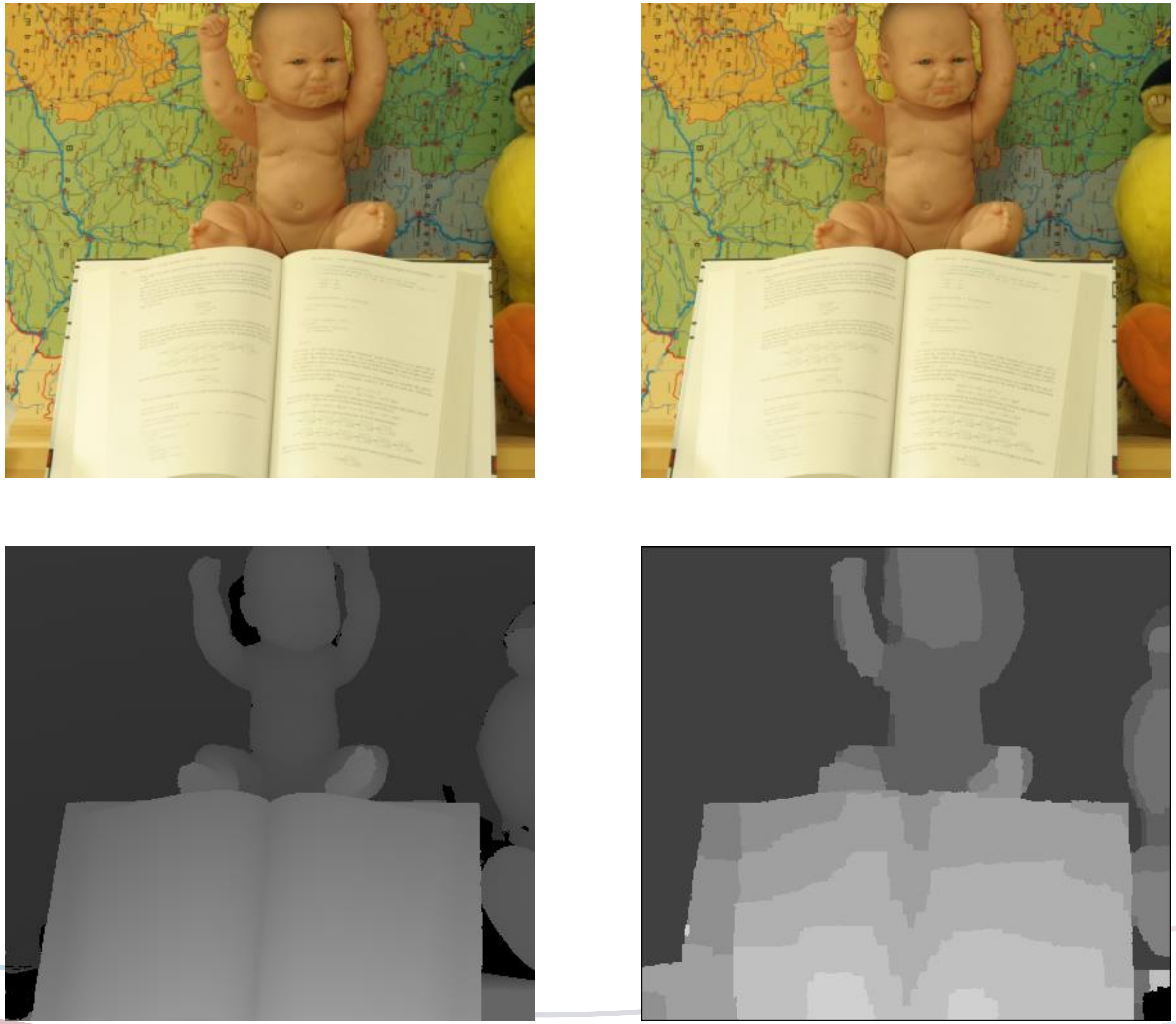




** middlebury 2006 stereo dataset*

# Results

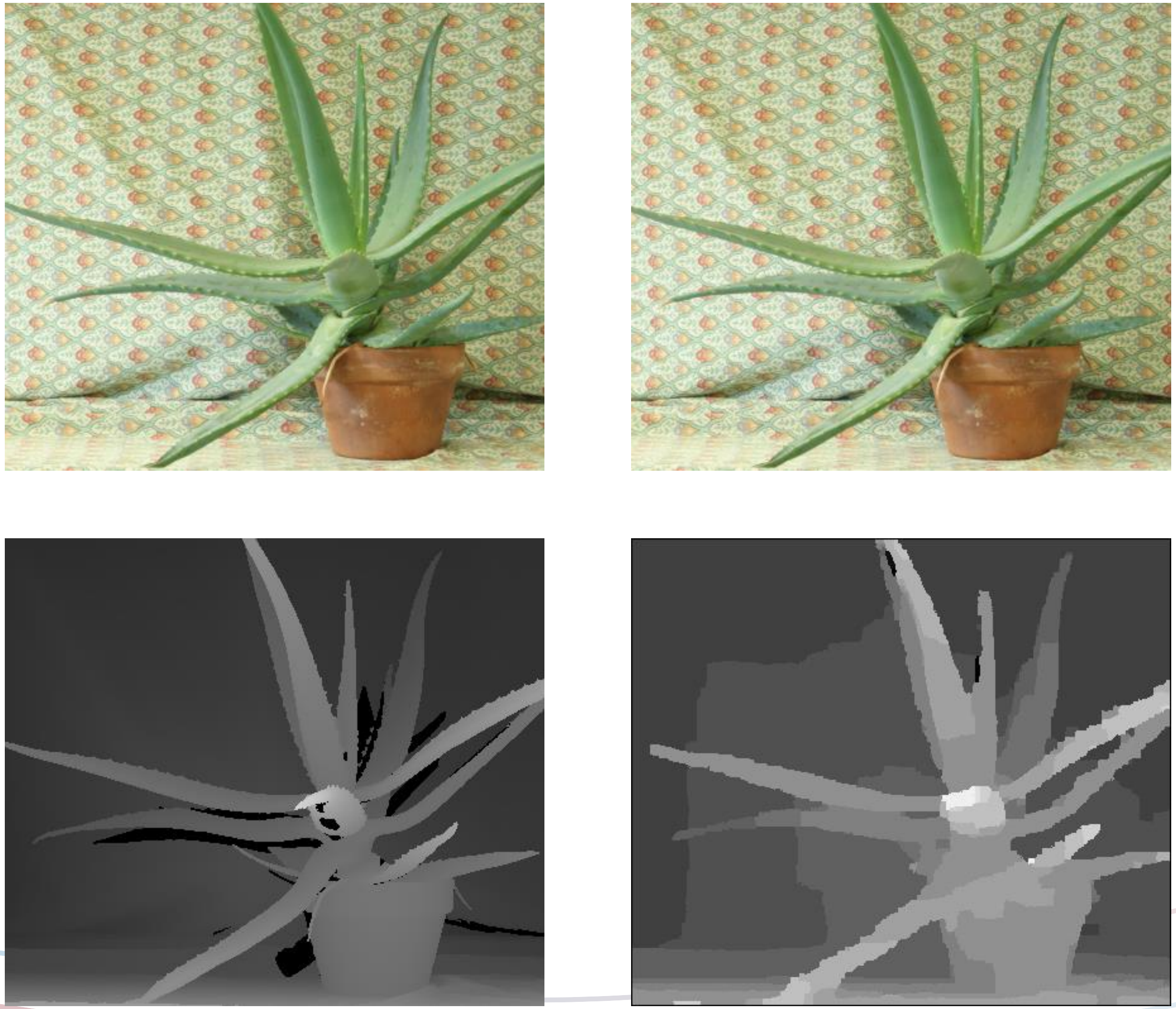
- baby2*



* middlebury 2006 stereo dataset

# Results

- aloe*



* middlebury 2006 stereo dataset

# Q & A

Thank You!!

# Belief Propagation

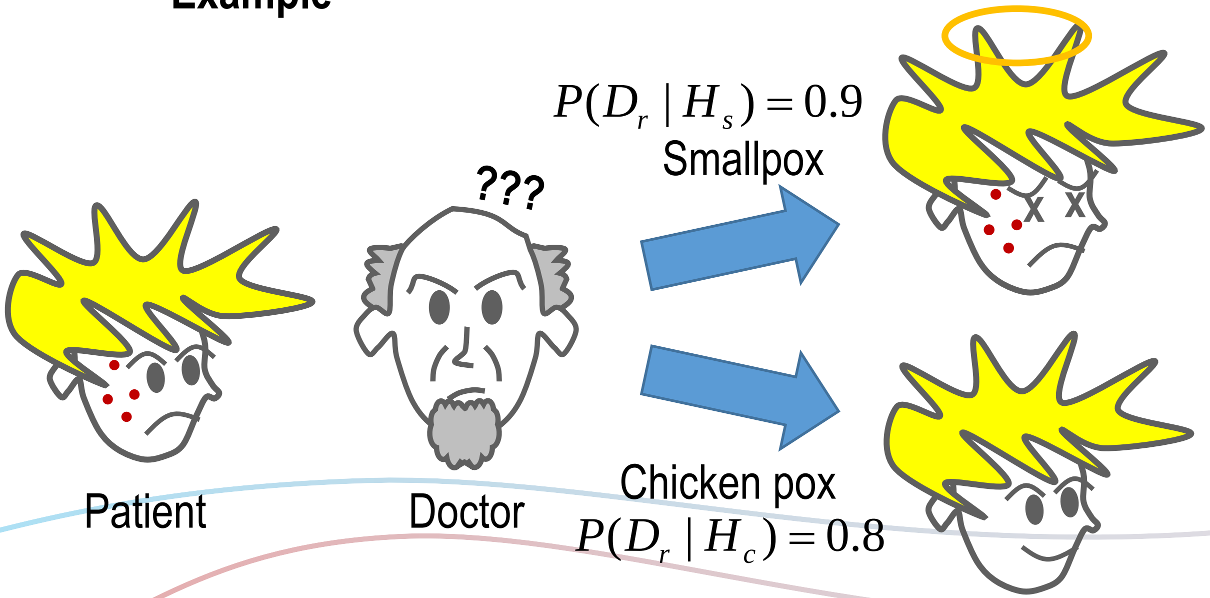
## ❖ Bayesian Probability (Bayesian Inference)

$$P(H \mid D) = \frac{P(H \cap D)}{P(D)} \qquad P(D \mid H) = \frac{P(D \cap H)}{P(H)}$$

$$P(H \cap D) = P(D \cap H) = P(D \mid H)P(H)$$

$$P(H \mid D) = \frac{P(D \mid H)P(H)}{P(D)}$$

**Example**




$$P(H_c \mid D_r) = \frac{P(D_r \mid H_c)P(H_c)}{H(D_r)}$$

$$P(D_r): \frac{81}{1000} = 0.081$$

$$P(H_c): \frac{1}{10} = 0.1$$

$$P(H_s): \frac{1}{1000} = 0.001$$

$$P(H_c \mid D_r) = \frac{0.8 \times 0.1}{0.081} = 0.988$$

$$P(H_s \mid D_r) = \frac{0.9 \times 0.001}{0.081} = 0.011$$